Non-stationary random fields as a combination of stationary random fields

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- Host institution: École Nationale Supérieure des Mines de Saint-Étienne, France
- Starting march. 2019, for 6 months.
- Salary: 1.150 €per months (minimum french salary SMIC).

Successfull candidates will have a strong background in applied or theoretical mathematics, with good skills in computer programming. Send CV/Resume and motivation letter to gavet@emse.fr.

Master thesis proposal

The core of the GEOFIELD project is to understand the geometry of random fields (denoted RF), in order to obtain practical tools for modeling and simulating actual spatial structures. Roughly speaking, a random field is denoted by a random variable in each physical point of a spatial domain and the correlation structure between neighbors. In Fig. 1 an example of a piecewise stationary Gaussian random field with Gaussian covariance is shown. These random objects are practical when modeling real structures like the surface of hip implants [1], human corneal endotheliums [4] and fuel cells [3]. This project focuses on Gaussian RFs because they are fully characterized by their covariance function.

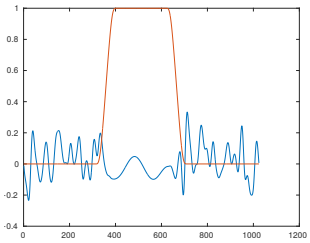

Figure 1: Non-stationary 1D RF simulated by combining 2 independent Gaussian RFs and smooth functions with small support.

For stationary Gaussian random fields with given covariance structure, a fairly standard method using the Fourier transform can efficiently speed-up the process [2]. To extend this efficient approach for non-stationary RFs, one could consider a RF G constructed by

$$G(x) = \sum_{i=1}^{n} G_i(x) f_i(x)$$

where $(G_i, i = 1, \ldots, N)$ is a sequence of independent, stationary Gaussian RFs with covariance function given by $C_{G_i}(x, y) = C_{G_i}(x - y)$ on a domain $D \subset \mathbb{R}^d$ and $(f_i, i = 1, \ldots, N)$ is a sequence of sufficiently smooth functions on D. Then the covariance function of G satisfies:

$$C_G(x, y) = \sum_{i=1}^{N} C_{G_i}(x - y) f_i(x) f_i(y).$$

Objective

Select the appropriate window functions f_i and stationary covariance functions C_{G_i} to approximate a given covariance function C_G efficiently. These RFs will be used to develop simulation models of Ni-YSZ anodes for fuel cells.

References

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[2] A. Lang and J. Potthoff. Fast simulation of Gaussian random fields. *Monte Carlo Methods and Applications*, 17(3):195–214, 2011.

[3] H. Moussaoui, J. Laurencin, Y. Gavet, G. Delette, M. Hubert, P. Cloetens, T. L. Bihan, and J. Debayle. Stochastic geometrical modeling of solid oxide cells electrodes validated on 3d reconstructions. *Computational Materials Science*, 143:262 – 276, 2018.

[4] K. Rannou, E. Crouzet, C. Ronin, P. Guerrero, G. Thuret, P. Gain, J. Pinoli, and Y. Gavet. Comparison of Corneal Endothelial Mosaic According to the Age: The CorImMo 3D Project. *IRBM*, 37(2):124–130, 2016.